NEET | IIT-JEE | FOUNDATION
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JEE MAIN-2020

COMPUTER BASED TEST (CBT)

DATE : 05-09-2020 (SHIFT-1) | TIME : (9.00 am to 12.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART-A : PHYSICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

1. A bullet of mass 5 g, travelling with a speed of 210 m/s, strikes a fixed wooden target. One half of its kinetic energy is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is 0.030 cal (g –°C) (1 cal = 4.2 × 10^7 ergs) close to :

(1) 38.4°C (2) 83.3°C (3) 87.5°C (4) 119.2°C

Ans. (3)

Sol. As per given condition

$$\frac{1}{2}\times\frac{1}{2}mv^2=(ms\Delta T)_{bullet}$$

$$\Delta t=\frac{V^2}{4s}$$

$$=\frac{210\times 210}{4\times 4.2\times 0.3\times 1000}=87.5^{\circ}C$$

2. A galvanometer of resistance G is converted into a voltameter of range 0 – 1V by connecting a resistance R_1 in series with it. The additional resistance that should be connected in series with R_1 to increase the range of the voltmeter to 0 – 2V will be :

(1) R_1 + G (2) R_1 (3) R_1 – G (4) G

Ans. (1)

Sol. $i_g(R_1+G) = 1$

$i_g(R_1+G+R_2) = 2$

$$\frac{1}{R_1+G}(R_1+G+R_2)=2$$

$R_1 + G + R_2 = 2R_1 + 2G$

$R_2 = R_1 + G.$

3. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packed is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity] :

(1) $t=\frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$ (2) $t=\sqrt{\left(\frac{2h}{3g}\right)}$ (3) $t=3.4\sqrt{\left(\frac{h}{g}\right)}$ (4) $t=1.8\sqrt{\left(\frac{h}{g}\right)}$

Ans. (3)

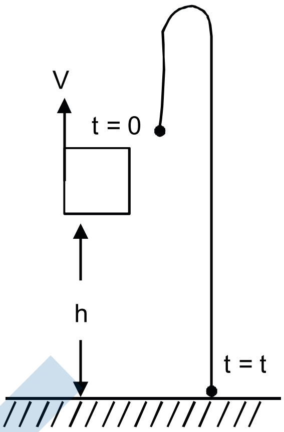
Sol. ⇒ For upward motion of helicopter

$v^2 = u^2 + 2as$

$v^2 = 0 + 2gh$

$v = \sqrt{2gh}$

⇒ Now particle will start moving under gravity.



$s = ut + \frac{1}{2}at^2$

$-h = \sqrt{2gh}\,t - \frac{1}{2}gt^2$

$\frac{1}{2}gt^2 - \sqrt{2gh}\,t - h = 0$

than $t = \dfrac{\sqrt{2gh} \pm \sqrt{2gh + 4\frac{g}{2} \times h}}{2 \times \frac{g}{2}}$

$t = \sqrt{\frac{2gh}{g}}(1+\sqrt{2})\ ;\ t = \sqrt{\frac{2h}{g}}(1+\sqrt{2})$

4. An electrical power line, having a total resistance of 2Ω, delivers 1 kW at 220 V. The efficiency of the transmission line is approximately :

(1) 85% (2) 91% (3) 96% (4) 72%

Ans. (3)

Sol. $i = \frac{P}{V} = \frac{1000}{220}$

$P_R = (i^2)R$

$\eta = \frac{1000 \times 100}{1000 + 41.32} = 96\%$.

5. A square loop of side 2a, and carrying current I, is kept in XZ plane with its centre at origin. A long wire carrying the same current I is placed parallel to the z-axis and passing through the point (0, b, 0), (b > > a). The magnitude of the torque on the loop about z-axis is given by:

(1) $\frac{2\mu_0 I^2 a^2}{\pi b}$ (2) $\frac{\mu_0 I^2 a^2}{2\pi b}$ (3) $\frac{2\mu_0 I^2 a^3}{\pi b}$ (4) $\frac{\mu_0 I^2 a^3}{2\pi b}$

Ans. (1)

Sol. $B = \frac{\mu_0 I}{2\pi b}$

Torque = τ = MBsinθ

$= \left[I_1(2a)^2\right]\left(\frac{\mu_0 I_2}{2\pi d}\right)\sin 90^\circ = \frac{2\mu_0 I_1 I_2}{\pi d} \times a^2$

$= \frac{2\mu_0 i^2 a^2}{\pi d}$.

6. The value of the acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface the earth. The ratio $\left(\frac{d}{R}\right)$ equals:

(1) $\frac{4}{9}$ (2) $\frac{1}{3}$ (3) $\frac{5}{9}$ (4) $\frac{7}{9}$

Ans. (3)

Sol. Given that

$g_n = g_d$

$\frac{GM}{(R+h)^2} = \frac{GM}{R^3}(R-d)$

$\frac{GM}{(R+R/2)^2} = \frac{GM}{R^3}(R-d)$

$\frac{4GM}{9R^2} = \frac{GM}{R^2}\left(1-\frac{d}{R}\right)$

$\frac{4}{9} = 1 - \frac{d}{R}$

$\frac{d}{R} = 1 - \frac{4}{9} = \frac{5}{9}$

$d = \frac{5}{9}R,$

7. Assume that the displacement (s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement (s) further depends on the speed of sound (ν), density of air (ρ) and the frequency (f). If $\Delta p \sim 10$Pa, n~300 m/s, p~1 kg/m^3 and f~1000 Hz, then s will be of the order of (take the multiplicative constant to be 1)

(1) 1 mm (2) $\frac{3}{100}$mm (3) 10 mm (4) $\frac{1}{10}$mm

Ans. (2)

Sol. $S = \frac{P_0}{BK}$

$= \frac{P_0}{PV^2 \frac{\omega}{V}}$

$= \frac{P_0}{PV\omega}$

$= \frac{10}{1 \times 1000 \times 300}$m

$\Rightarrow \frac{3}{100}$ m.

8. A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia $3I$ initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is:

(1) $\frac{3}{4}$ (2) $\frac{1}{4}$ (3) 0 (4) $\frac{5}{6}$

Ans. (1)

Sol. From angular momentum conservation

$I\omega + 0 = I\omega_C + 3I\omega_C$

$\omega_C = \frac{\omega}{4}$

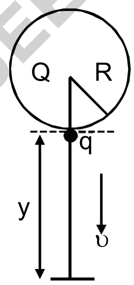
Loss of kinetic energy = $\frac{1}{2}I\omega^2 - \frac{1}{2}(I+3I)\left(\frac{\omega}{4}\right)^2$

$= \frac{1}{2}I\omega^2 - \frac{1}{2}I\frac{\omega^2}{4}$

$= \frac{3}{8}I\omega^2$

Fractional loss = $\frac{3}{4}$.

9. A solid sphere of radius R carries a charge Q + q distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then (assume the remaining portion to be spherical)



(1) $v^2 = 2y\left[\frac{QqR}{4\pi\varepsilon_0(R+y)^3 m} + g\right]$

(2) $v^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R^2 ym} + g\right]$

(3) $v^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

(4) $v^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

Ans. (4)

Sol. By using total energy conservation

$\Delta KE + (\Delta PE)_{Electo} + (\Delta PE)_{gravitationl} = 0$

$$\frac{1}{2}mV^2 + \left(k\frac{Qq}{R+y} - k\frac{Qq}{R}\right) + (-mgy) = 0$$

$$\frac{1}{2}mV^2 = mgy + kQq\left(\frac{1}{R} - \frac{1}{R+y}\right) \; ; \; V^2 = 2gy + \frac{2kQq}{m}\frac{y}{R(R+y)}$$

$$V^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$$

10. A physical quantity z depends on four observables a, b, c and d, as $z = \frac{a^2 b^{2/3}}{\sqrt{cd^3}}$. The percentage of error in the measurement of a,b,c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is :

(1) 13.5% (2) 16.5% (3) 14.5% (4) 12.25%

Ans. (3)

Sol. $\frac{\Delta t}{t} = \frac{2\Delta a}{a} + \frac{2}{3}\frac{\Delta c}{b} + \frac{1}{2}\frac{\Delta c}{c} + 3\frac{\Delta d}{d}$

$= 2 \times 2 + \frac{2}{3} \times 1.5 + \frac{1}{2} \times 4 + 3 \times 2.5$

= 4 + 1 + 2 + 7.5

= 14.5%

11. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to ? (Given, mean kinetic energy of a molecule (at T) is 4×10^{-14} erg, g = 980 $cm/^2$, density of mercury = 13.6 g/cm^3).

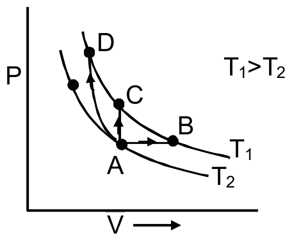
(1) 5.8×10^{16} (2) 4.0×10^{16} (3) 4.0×10^{18} (4) 5.8×10^{18}

Ans. (3)

Sol. $N = \frac{PT}{KT}$

U = $U = \frac{3}{2}KT$ $\therefore$ $N = \frac{3PV}{2U} = 3.99 \times 10^{18}$

12. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are labelled as A → B, A → C and A → D. The change in internal energies during these process are taken as E_{AB}, E_{AC} and E_{AD} and the work done as W_{AB}, W_{AC} and W_{AD}. The correct relation between these parameters are:



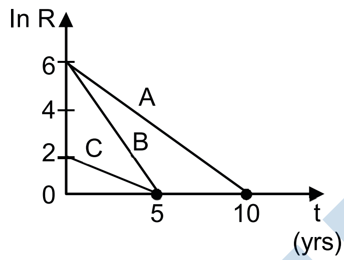
(1) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$
(2) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} > 0$
(3) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$
(4) $E_{AB} = E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$

Ans. (2)

Sol. ΔT is same for $E_{AB} = E_{AC} = E_{AD}$

Note : In second option $W_{AD} < 0$ but it is correct option is NTA.

13. Activities of three radioactive substances A, B and C are represented by the curves A, B and C, in the figure. Then their half-lives $T_{\frac{1}{2}}(A) : T_{\frac{1}{2}}(B) : T_{\frac{1}{2}}(C)$ are in the ratio :



(1) 2 : 1 : 3 (2) 3 : 2 : 1 (3) 4 : 3 : 1 (4) 2 : 1 : 1

Ans. (1)

Sol. $R = R_0 e^{-\lambda t}$

$\ell nR = -\lambda \ell nt + \ell nR_0$

Slope = $\frac{\ell n2}{t_{1/2}} = \lambda$

$T_{\frac{1}{2}}(A) : T_{\frac{1}{2}}(B) : T_{\frac{1}{2}}(C) = 2:1:3.$

14. In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the running fork frequency is :

(1) 2200 Hz (2) 1100 Hz (3) 3300 Hz (4) 550 Hz

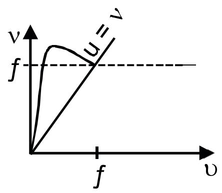
Ans. (1)

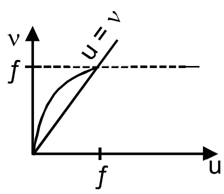
Sol. $\frac{\lambda}{2}$ = 24.5 – 17 = 7.5 cm

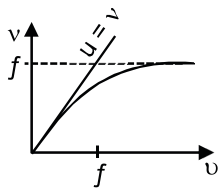
$f = \frac{V}{\lambda}$

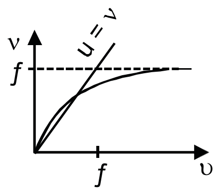
$= \frac{330 \times 100}{15} = 2200\text{Hz}.$

15. For a concave lens of focal length f, the relation between object and image distance μ and ν, respectively, from its pole can best the represented by ($\upsilon = \nu$ is the reference line) :

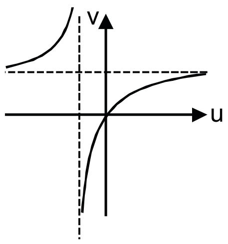
(1) 

(2) 

(3) 

(4) 

Ans. (3)

Sol. 

16. With increasing biasing voltage of a photodiode, the photocurrent magnitude :

(1) remains constant

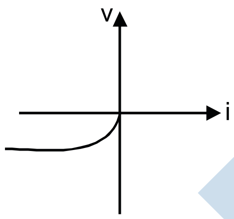
(2) increases initially and saturates finally

(3) increases linearly

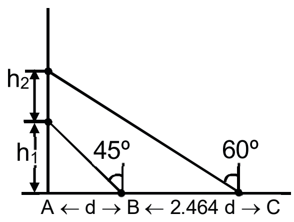
(4) increases initially and after attaining certain value, it decreases

Ans. (2)

Sol. Theory based



17. A balloon is moving up in are vertically above a point A on the ground. When it is a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan30° = 0.5774):



(1) 0.732 d (2) 0.464 d (3) 1.464 d (4) d

Ans. (4)

Sol. $t_{45} = \frac{h_1}{d}$

$h_1 = d$ ....(i)

$\frac{h_1 + h_2}{d + 2.464d} = \tan 30° = \frac{1}{\sqrt{3}} = 0.5774$

$h_1 + h_2 = (3.464)d \times 0.5774$

$h_1 + h_2 = 2d$ ....(ii)

$\therefore \quad h_2 = d.$

18. A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ with respect to water, the value of r is :

(1) $\frac{1}{3}R$ (2) $\frac{4}{9}R$ (3) $\frac{2}{3}R$ (4) $\frac{8}{9}R$

Ans. (4)

Sol. In equilibrium

$mg = F_B$

$\frac{4}{3}\pi(R^3 - r^3)\rho_0 g = \frac{4}{3}\pi R^3 \rho_w g$

$\left[1 - \left(\frac{r}{R}\right)^3\right]\frac{27}{8}\rho_w = \rho_w$

$1 - \frac{r^3}{R^3} = \frac{9}{27}$

$1 - \frac{1}{3} = \frac{r^3}{R^3}$

$\frac{2}{3} = \frac{r^3}{R^3}$

$\frac{r}{R} = \left(\frac{2}{3}\right)^{1/3}$

$1 - \frac{r^3}{R^3} = \frac{8}{27}$

$\frac{r^3}{R^3} = 1 - \frac{8}{27} = \frac{19}{27}$

r = 0.89.

19. An electron is constrained to move along the y-axis with a speed of 0.1c (c is the speed of light) in the presence of electromagnetic wave, whose electric field is $\vec{E} = 30\hat{j}\ \sin(1.5 \times 10^7 t - 5 \times 10^{-2} x)$ V/m. The maximum magnetic force experience by the electron will be:

(given $c = 3 \times 10^8\ ms^{-1}$ and electron charge $= 1.6 \times 10^{-19}C$)

(1) 2.4×10^{-18} N (2) 1.6×10^{-19} N (3) 4.8×10^{-19} N (4) 3.2×10^{-18} N

Ans. (3)

Sol. In electromagnetic wave is $\frac{E_0}{B_0} = C$

so maximum value of magnetic field is

$$B_0 = \frac{E_0}{C}$$

$F_{max.} = qVB_{max}.\sin 90^o$

$$= \frac{qV_0E_0}{C}$$

$$\frac{1.6 \times 10^{-19} \times 0.1 \times 3 \times 10^8 \times 30}{3 \times 10^8} = 4.8 \times 10^{-19}\,N$$

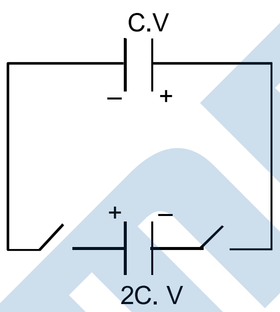
20. Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is :

(1) $\frac{25}{6}CV^2$ (2) $\frac{9}{2}CV^2$ (3) zero (4) $\frac{3}{2}CV^2$

Ans. (4)

Sol.

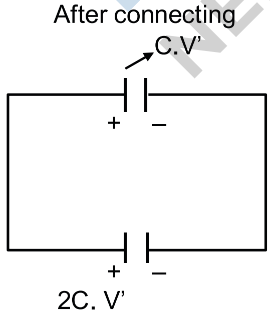
Before



Charge on C = CV

Charge on 2C = (2C) 2V)

After connecting



From charge conservation, $2C(2V) - CV = (C + 2C)V'$

Common potential $V' = V$

$$U_f = \left(\frac{1}{2}CV^2 + \frac{1}{2} \times 2CV^2\right) = \frac{3}{2}CV^2$$

$\Delta U = 3CV^2$

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

21. A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})N$ acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})m$. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})m$ will be $\sqrt{x}\,N-m$. The value of x is……….

Ans. 195

Sol. $\vec{r} = (4-1)\hat{i} + (3-2)\hat{j} + (-1-1)\hat{k}$

$= 3\hat{i} + \hat{j} - 2\hat{k}$

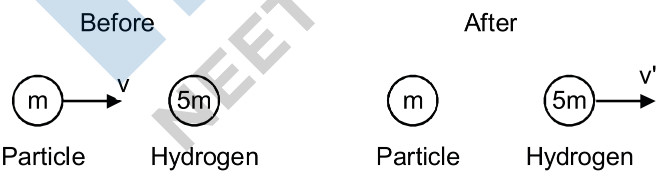
$$\tau = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix}$$

$= \hat{i}(7) - \hat{j}(11) + \hat{k}(5) = 7\hat{i} - 11\hat{j} + 5\hat{k} = \sqrt{49 + 121 + 25} = \sqrt{195}$

22. A particle of mass 200 MeV/c^2 collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $\frac{N}{4}$. The value of N is: (Given the mass of the hydrogen atom to be 1 GeV/c^2)……..

Ans. 51

Sol.



For linear momentum conservation

$mV + 0 = 0 + 5mV'$

$$V' = \frac{v}{5}$$

loss of KE $= \frac{1}{2}mv^2 - \frac{1}{2}(5m)\left(\frac{v}{5}\right)^2 = \frac{1}{2}mv^2\left(1-\frac{1}{5}\right)$

$= \frac{4}{5}\left(\frac{mv^2}{2}\right) = \frac{4}{5}k = 10.2\text{ eV}$

$k = 12.75\text{eV} = \frac{N}{4}$

N = 51.

23. Two concentric circular coils, C_1 and C_2, are placed in the XY plane. C_1 has 500 turns, and a radius of 1 cm. C_2 has 200 turns and radius of 20 cm. C_2 carries a time dependent current $I(t) = (5t^2 - 2t + 3)$ A where t is in s. The emf induced in C_1 (in mV), at the instant t 1s is $\frac{4}{x}$. The value of x is …….

Ans. 5

Sol. $I = (5t^2 + 2t + C)$

$\frac{di}{dt} = (10t+2)$

$\phi_{small} = BA = \left(\frac{\mu_0 I N_2}{2R}\right)(\pi r^2)$

induced emf in small coil

$e = \frac{d\phi}{dt} = \left(\frac{\mu_0 N_2}{2r}\right)\pi r^2 N_1 \frac{di}{dt} = \left(\frac{\mu_0 N_1 N_2 \pi r^2}{2R}\right)(10t-2)$

at t = 1

$e = \left(\frac{\mu_0 N_1 N_2 \pi r^2}{2R}\right)8 = 4\frac{\mu_0 N_1 N_2 \pi r^2}{R} = \frac{4(4\pi)10^{-7} \times 200}{20} \times 500 \times \frac{10^{-4}}{10^{-2}}\pi$

$= 80 \times \pi^2 \times 10^{-7} \times 10 \times 10^2 \times 10^{-2} = 8 \times 10^{-4}$ volt $= 0.8$ mV $= \frac{4}{x}$

x = 5.

24. A beam of electrons of energy E scatters from a target having atomic spacing of 1Å. The first maximum intensity occurs at $\theta = 60°$. Then E (in eV) is……..

(Plank constant h = 6.64 × 10–34 Js, 1 eV = 1.6×10^{-19} J, electron mass m = 9.1 × 10–31 kg).

Ans. 50

Sol. $2d\sin\theta = n\lambda$

$2d\frac{\sqrt{3}}{2} = (1)\lambda$

d = 1Å

$\lambda = \sqrt{3}$Å

$\sqrt{3} = \sqrt{\frac{150}{v}}$

V = 50 volt

E = 50 eV.

25. A compound microscope consists of an objective lens of focal length 1 cm and an eye piece of focal length 5 cm with a separation of 10 cm. The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of n is............

Ans. 50

Sol. L = 10

$v_e = \infty$

$u_e = f_e = 5$

$v_0 = 10 - 5 = 5$

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$$

$$\frac{1}{5} - \frac{1}{u_0} = \frac{1}{1}$$

$$u_0 = -\frac{5}{4}\text{cm}$$

$$\frac{5}{4} = \frac{n}{40}$$

n = 50.

PART-B : CHEMISTRY

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

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26. Consider the following reaction :

$$N_2O_4 \rightleftharpoons 2NO_2\,(g)\ ;\ \Delta H^o = +58\ kJ$$

For each of the following cases (a,b), the direction in which the equilibrium shifts is :

(a) Temperature is decreased.

(b) Pressure is increased by adding N_2 at constant T.

(1) (a) towards reactant, (b) no change

(2) (a) towards product, (b) no change

(3) (a) towards product, (b) towards reactant

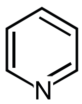
(4) (a) towards reactant (b) towards product

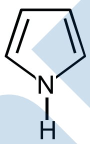
Ans. (1)

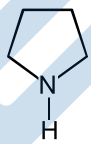
Sol. (i) As reaction is endothermic so on decrease in temperature equilibrium shift in reactant side.

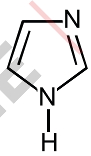
(ii) On increase in pressure by adding inert gas at same temperature, no shifting will take place

27. The increasing order of basicity of the following compounds is :


(A)

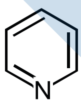

(B)

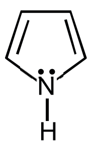

(C)

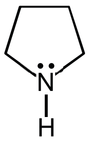

(D)

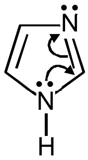
(1) (B) < (A) < (C) < (D)
(2) (A) < (B) < (C) < (D)
(3) (D) < (A) < (B) < (C)
(4) (B) < (A) < (D) < (C)

Ans. (4)

Sol. (A)  Nitrogen atom is sp^2 hybridised and lone pair is localized

(B)  Nitrogen atom is sp^2 hybridised and lone pair is delocalized

(C)  Nitrogen atom is sp^3 hybridised and lone pair is localized

(D)  Nitrogen atom is sp^2 hybridised and lone pair is localised with partial negative charge.

28. If a person is suffering from the deficiency of nor-adrenaline, what king of drug can be suggested ?

(1) Antihistamine (2) Analgesic (3) Anti-inflammatory (4) Antidepressant

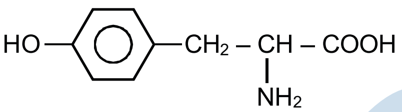
Ans. (4)

Sol. If the level of noradrenaline is low for some reason, then the signal-sending activity becomes low, and the person suffers from depression. In such situations, antidepressant drugs are required.

29. Which of the following is not an essential amino acid ?

(1) Tyrosine (2) Valine (3) Leucine (4) Lysine

Ans. (1)

Sol. Tyrosine  is a non-essential amino acid.

30. An Ellingham diagram provides information about :

(1) the kinetics of the reduction process.

(2) the conditions of pH and potential under which a species is thermodynamically stable.

(3) the pressure dependence of the standard electrode potentials of reduction reactions involved in the extraction of metals.

(4) the temperature dependence of the standard Gibbs energies of formation of some metal oxides.

Ans. (4)

Sol. Ellingham diagram is graph of ΔG^o vs T of any metal / element oxide. Since

$\Delta G^o = \Delta H^o - T\Delta S^o$

for most metal oxide formation

metal (s) + oxygen (g) $\rightarrow$ metal oxide (s)

$\Delta H^o = -$ ve

$\Delta S^o = -$ ve

so graph will be a straight line with – ve y- intracept & +ve slope.

31. The condition that indicates a polluted environment is :

(1) 0.03% of CO_2 in the atmosphere (2) eutrophication

(3) pH of rain water to be 5.6 (4) BOD value of 5 ppm

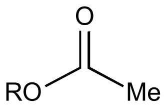
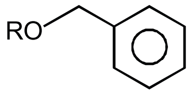
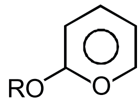
Ans. (2)

Sol. (2) Clean water would have B.O.D value of less than 5 ppm whereas highly polluted water coluld have a B.O.D value of 17 ppm or more.

(3) The process in which nutrient enriched water bodies support a dense plant population which kill animal life by depriving it of oxygen results in subsequent loss of biodiversity is known as Eutrophication.

(4) If the concentration of dissolved oxygen of water is below 6 ppm, the growth of fish get inhibited.

32. Which of the following derivatives of alcohols is unstable in an aqueous base?

(1)  (2) RO – CMe_3 (3)  (4) 

Ans. (1)

Sol. Esters are hydrolysed in basic medium(saponification)

33. The equation that represents the water-gas shift reaction is :

(1) $CO(g) + H_2O(g) \xrightarrow[\text{Catalyst}]{673K} CO_2(g) + H_2(g)$

(2) $2C(g) + O_2(g) + 4N_2(g) \xrightarrow{1273K} 2CO(g) + 4N_2(g)$

(3) $C(g) + H_2O(g) \xrightarrow{1270K} CO(g) + H_2(g)$

(4) $CH_4(g) + H_2O(g) \xrightarrow[\text{Ni}]{1270K} CO(g) + 3H_2(g)$

Ans. (1)

Sol. $CO(g) + H_2O(g) \xrightarrow[\text{Catalyst}]{673K} CO_2(g) + H_2(g)$

Reaction is called water gas shift reaction

34. A flask contains a mixture of compounds A and B. Both compounds decompose by first-order kinetics. The half-lives for A and B are 300 s and 180 s, respectively. If the concentrations of A and B are equal initially, the time required for the concentration of A to be four times that of B (in s) is : (use ln 2 = 0.693)

(1) 120 (2) 300 (3) 180 (4) 900

Ans. (4)

Sol. $C_t = C_0 e^{-kt}$ $k_A = \frac{\ln 2}{180}$ $\left(k = \frac{\ln 2}{T_{1/2}}\right)$

$(C_t)_A = (C_0)_A e^{-k_A t}$ $k_B = \frac{\ln 2}{300}$

$(C_t)_B = (C_0)_B e^{-k_B t}$

$$\frac{(C_t)_B}{(C_t)_A} = \frac{(C_0)_B}{(C_0)_A} \times e^{(k_A - k_B)t}$$

$$4 = e^{(k_A - k_B)t}$$

$$2\ln 2 = \left[\frac{\ln 2}{180} - \frac{\ln 2}{300}\right]t$$

$$2 = \left(\frac{120}{180 \times 300}\right)t$$

$$t = \frac{2 \times 180 \times 300}{120} = 900 \text{ sec}$$

35. The value of the crystal field stabilization energies for a high spin d6 metal ion in octahedral and tetrahedral fields, respectively, are:

(1) $-2.4\Delta_0$ and $-0.6\Delta_t$ (2) $-1.6\Delta_0$ and $-0.4\Delta_t$

(3) $-0.4\Delta_0$ and $-0.27\Delta_t$ (4) $-0.4\Delta_0$ and $-0.6\Delta_t$

Ans. (4)

Sol. For d^6 configuration, high spin

(i) In case of octahedral complex

$t_{2g}^{2,1,1}$, $e_g^{1,1}$

CFSE = $[-0.4n_{t_{2g}} + 0.6n_{eg}]\Delta_0 + n(P)$

$= [-0.4 \times 4 + 0.6 \times 2]\Delta_0 + 0$

$= -0.4\Delta_0$

(i) In case of tetrahedral complex

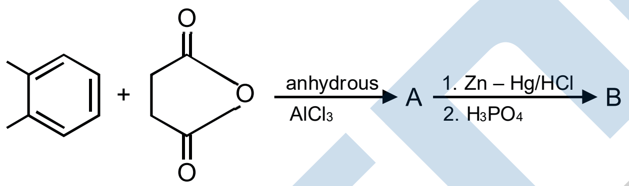
$e^{2,1}$, $t_2^{1,1,1}$

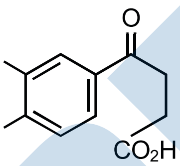
CFSE = $[-0.6n_e + 0.4n_{t_2}]\Delta t$

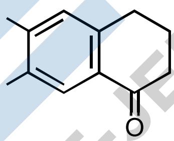
$= [-0.6 \times 3 + 0.4 \times 3]\Delta t$

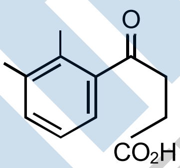
$= -0.6\Delta t$

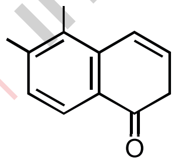
36. In the following reaction sequence the major product A and B are :

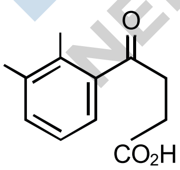


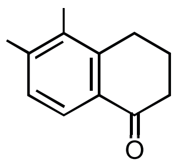
(1) A = 

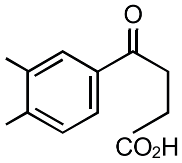
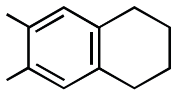
; B = 

(2) A = 

; B = 

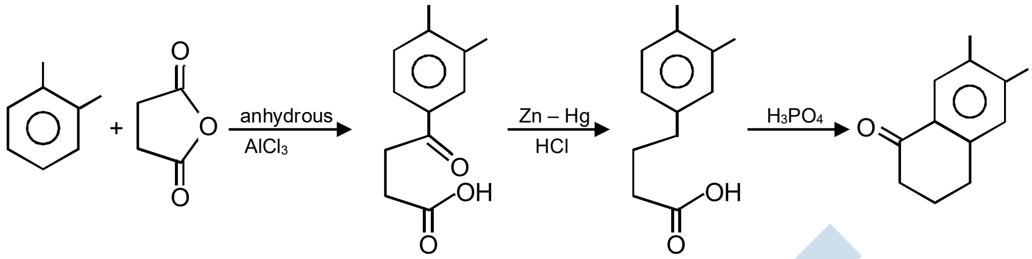
(3) A = 

; B = 

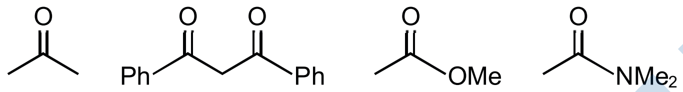
(4) A =  ; B = 

Ans. (1)

Sol.



37. The increasing order of the acidity of the α-hydrogen of the following compounds is:

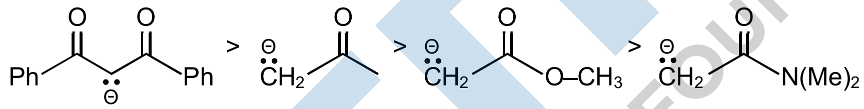


(A) (B) (C) (D)

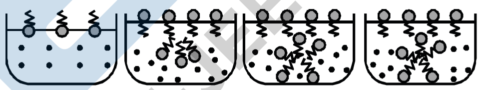
(1) (C) < (A) < (B) < (D)
(2) (B) < (C) < (A) < (D)
(3) (A) < (C) < (D) < (B)
(4) (D) < (C) < (A) < (B)

Ans. (4)

Sol. Acidity α stability of conjugate base



38. Identify the correct molecular picture showing what happens at the critical micellar concentration (CMC) of an aqueous solution of a surfactant (polar head; non-polar tail water)



(A) (B) (C) (D)

(1) (A)
(2) (B)
(3) (D)
(4) (C)

Ans. (3)

Sol. In micelle formation, above "CMC" hydrocarbon chains are pointing towards the centre of sphere with COO^- part remaining outward on the surface.

39. In the sixth period, the orbitals that are filled are:

(1) 6s, 5f, 6d, 6p
(2) 6s, 5d, 5f, 6p
(3) 6s, 6p, 6d, 6f
(4) 6s, 4f, 5d, 6p

Ans. (4)

Sol. In 6^{th} period 6s, 4f, 5d and 6p orbitals are gradually filled.

40. A diatomic molecule X_2 has a body-centred cubic (bcc) structure with a cell edge of 300 pm. The density of the molecule is 6.47 g cm^{-3}. The number of molecules present in 200 g of X_2 is:

(1) $4N_A$
(2) $2N_A$
(3) $40N_A$
(4) $8N_A$

Ans. (1)

Sol. For BCC Z = 2

$$d = \frac{Z \times M}{N_A \times \text{Volume}}$$

$$6.17 = \frac{2 \times M}{6.02 \times 10^{23} \times [3 \times 10^{-8}]^3}$$

$$6.17 = \frac{2 \times M}{6.02 \times 2.7}$$

M = 50

$$\text{No. of mol} = \frac{200}{50} = 4$$

No. of molecule = 4 N_A

41. The structure of PCl_5 in the solid state is :

(1) tetrahedral $[PCl_4]^+$ and octahedral $[PCl_6]^-$ (2) trigonal bipyramidal

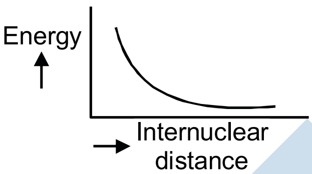
(3) square planar $[PCl_4]^+$ and octahedral $[PCl_6]^-$ (4) square pyramidal

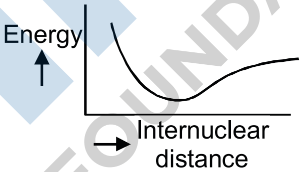
Ans. (1)

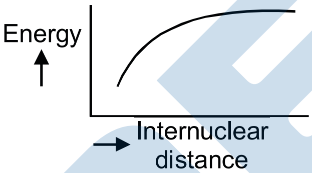
Sol. $2PCl_5(s) \longrightarrow [PCl_4]^+ \quad [PCl_6]^-$

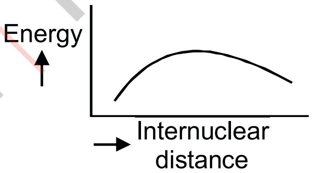
Tetrahedral Octahedral

42. The potential energy curve for the H_2 molecule as a function of internuclear distance is:

(1)


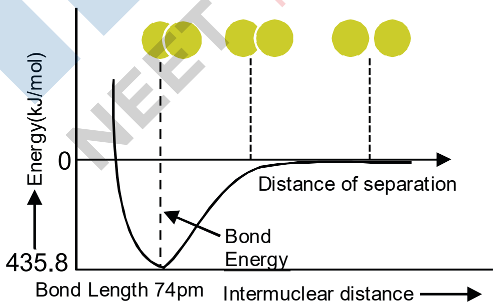
(2)


(3)


(4)


Ans. (2)

Sol. The potential energy curve for the formation of H_2 molecule as a function of internuclear distance of the H atoms. The minima in the curve corresponds to the most stable state of H_2.



43. The correct electronic configuration and spin-only magnetic moment (BM) of Gd^{3+} (Z=64), respectively, are:

(1) [Xe] $5f^7$ and 7.9 (2) [Xe] $5f^7$ and 8.9 (3) [Xe] $4f^7$ and 7.9 (4) [Xe] $4f^7$ and 8.9

Ans. (3)

Sol. Electronic configuration of

${}_{64}Gd$ = [Xe] $4F^7\ 5d^1\ 6s^2$

${}_{64}Gd^{3+}$ = [Xe] $4F^7$

No. of unpaired electron = 7

$\mu = \sqrt{n(n+2)}\,BM$

$= \sqrt{63}$

= 7.93 BM

44. The difference between the radii of 3rd and 4th orbitals of Li^{2+} is ΔR_1. The difference between the radii of 3rd and 4th orbits of He^+ is ΔR_2. Ratio $\Delta R_1 : \Delta R_2$ is :

(1) 2 : 3 (2) 3 : 8 (3) 3 : 2 (4) 8 : 3

Ans. (1)

Sol. $r = 0.529\frac{n^2}{Z}\text{Å}$

For Li^{2+}

$(r_{Li^{2+}})_{n-4} - (r_{Li^{2+}})_{n-3} = \frac{0.529}{3}[16-9] = \Delta R_1$

For He^+

$(r_{He^+})_{n-4} - (r_{He^+})_{n-3} = \frac{0.529}{2}[16-9] = \Delta R_2$

$\frac{\Delta R_1}{\Delta R_2} = \frac{2}{3}$

45. The most appropriate reagent for conversion of C_2H_5CN into $CH_3CH_2CH_2NH_2$ is :

(1) $NaBH_4$ (2) $LiAlH_4$ (3) CaH_2 (4) $Na(CN)BH_3$

Ans. (2)

Sol. $CH_3CH_2-C\equiv N \xrightarrow{LiAlH_4} CH_3CH_2-CH_2-NH_2$

$NaBH_4$ does not reduce R–CN.

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

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46. An oxidation-reduction reaction in which 3 electrons are transferred has a ΔG^o of 17.37 kJ mol^{-1} at 25ºC. The value of E^0_{cell} (in V) is ……….× 10^{-2}. (1F = 96,500 C mol^{-1})

Ans. –6

Sol. $\Delta G^\circ = -nFE^0_{cell}$

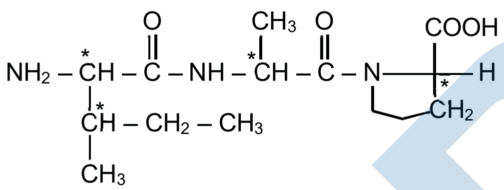
$17.37 \times 10^3 = -3 \times 96500 \times E^0_{cell}$

$E_{cell} = -\ 0.06$ V

47. The number of chiral carbon(s) present in peptide, Ile-Arg-Pro, is…….

Ans. 4

Sol.



48. The minimum number of moles of O_2 required for complete combustion of 1 mol of propane and 2 moles of butane is…………

Ans. 18

Sol. (1) Combustion of propane.

$$\underset{1\,mol}{C_3H_8} + \underset{5\,mol}{5O_2(g)} \longrightarrow 3CO_2(g) + 4H_2O$$

(2) Combustion of butane

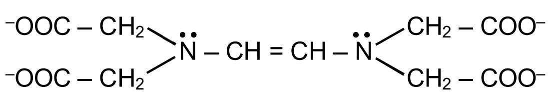
$$\underset{1\,mol}{C_4H_{10}} + \underset{\frac{13}{2}\,mol}{\frac{13}{2}O_2} \longrightarrow 4CO_2(g) + 5H_2O$$

so total mol of O_2 required = 5 + 13 = 18

49. The total number of coordination sites in ethylene di aminetetra acetate ($EDTA^{4-}$) is…….

Ans. 6

Sol.



50. A soft drink was bottled with a partial pressure of CO_2 of 3 bar over the liquid at room temperature. The partial pressure of CO_2 over the solution approaches a value of 30 bar when 44 g of CO_2 is dissolved in 1 kg of water at room temperature. The approximate pH of the soft drink is……. × 10^{-1}.

(First dissociation constant of H_2CO_3 = 4.0 × 10^{-7}; log2 = 0.3; density of the soft drink = 1 g mL^{-1})

Ans. 37

Sol. Amount of CO_2 in one liter of solution = 4.4. gram = 0.1 Mol

pH = 1/2 {pKa - log C} For a weak acid solution

pH = 1/2 { 6.4 +1 } = 3.7

PART-C : MATHEMATICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

51. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be :

(1) 36 (2) 63 (3)38 (4) 54

Ans. **(1)**

Sol. $n(C) = 73$, $n(T) = 65$, $n(C \cap T) = x$

$n(C \cup T) \le 100$

$\Rightarrow$ $n(C) + n(T) - n(C \cap T) \le 100$

$\Rightarrow$ $x \ge 38$

$n(C \cap T) \le \min(n(C), n(T))$ $\Rightarrow$ $x \le 65$

$\Rightarrow$ $38 \le x \le 65$

52. If the function f(x)= $\begin{cases} k_1(x-\pi)^2 - 1, & x \le \pi \\ k_2 \cos x, & x > \pi \end{cases}$ is twice differentiable, then the ordered pair (k_1, k_2) is equal to :

(1) $\left(\frac{1}{2}, -1\right)$ (2) $\left(\frac{1}{2}, 1\right)$ (3) (1, 0) (4) (1, 1)

Ans. (2)

Sol. f(x) is differentiable then will also continuous

then $f(\pi) = -1$, $f(\pi^{+1}) = -k_2$

$k_2 = 1$

Now

$$f'(x) = \begin{cases} 2k_1(x-\pi) & x \le \pi \\ -k_2 \sin x & x > \pi \end{cases}$$

then $f'(\pi^-) = f'(\pi^+) = 0$

$$f''(x) = \begin{cases} 2k_1 & x \le \pi \\ -k_2 \cos x & x > \pi \end{cases}$$

then $2k_1 = k_2$

$$k_1 = \frac{1}{2}$$

53. If $3^{2\sin 2\alpha-1}$, 14 and $3^{4-2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth term of this A.P. is

(1) 81 (2) 65 (3) 66 (4) 78

Ans. **(3)**

Sol. a, b, c are in AP then

2b = a + c

$28 = 3^{2\sin 2\theta-1} + 3^{4-2\sin 2\theta}$

Put $3^{2\sin 2\theta} = x$

$$28 = \frac{x}{3} + \frac{81}{x} \Rightarrow x^2 - 84x + 243 = 0$$

(x – 3) (x – 81) = 0

$3^{2\sin 2\theta} = 3$ or 3^4

$\sin 2\theta = 1$ or 4

$\sin 2\theta = \frac{1}{2}$

terms are 1, 14, 27, ………….. then $T_6 = 1 + 5\,(13)$

54. If S is the sum of the first 10 terms of the series $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \ldots,$ then tan(S) is equal to :

(1) $-\frac{6}{5}$ (2) $\frac{5}{11}$ (3) $\frac{5}{6}$ (4) $\frac{10}{11}$

Ans. (3)

Sol. $S = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \ldots\ldots$ upto 10 term

$$S = \tan^{-1}\left(\frac{2-1}{1+1.2}\right) + \tan^{-1}\left(\frac{3-2}{1+1.3}\right) + \tan^{-1}\left(\frac{4-3}{1+3.4}\right) + \ldots\ldots + \tan^{-1}\left(\frac{11-10}{1+11.10}\right)$$

$S = (\tan^{-1}2 - \tan^{-1}1) + (\tan^{-1}3 - \tan^{-1}2) + \ldots\ldots\ldots + (\tan^{-1}11 - \tan^{-1}10)$

$S = \tan^{-1}11 - \tan^{-1}1$

$$S = \tan^{-1}(11) - \frac{\pi}{4}$$

$$\tan S = \frac{5}{6}$$

55. If $2^{10} + 2^9.3^1 + 2^8.3^2 + \ldots.. + 2.3^9 + 3^{10} = 3^{10} = S - 2^{11}$ then S is equal to

(1) 3^{11} (2) 2.3^{11} (3) $\frac{3^{11}}{2} + 2^{10}$ (4) $3^{11} - 2^{12}$

Ans. **(1)**

Sol. $S' = 2^{10} + 2^9.3 + 2^8.3^2 + \ldots\ldots + 2\,.\,3^9 + 3^{10}$

$G.P. \rightarrow a = 2^{10}, r = \frac{3}{2}, n = 11$

$S' = 2^{10}.\frac{\left(\left(\frac{3}{2}\right)^{11} - 1\right)}{\frac{3}{2} - 1} = 2^{11}\left(\frac{3^{11}}{2^{11}} - 1\right)$

$= 3^{11} - 2^{11}$

56. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then c is equal to :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{2}$ (4) $\frac{1}{4}$

Ans. (1)

Sol. $y^2 = 4x$ & $x^2 = 4y$

any tangent of $y^2 = 4x$ is $y = mx + \frac{1}{m}$

it also tangent for $x^2 = 4y$

$\therefore \quad \frac{1}{m} = -m \Rightarrow \quad m = -1$

$\therefore$ common tangent is $y = -x - 1$, it also touches $x^2 + y^2 = c^2$

$\therefore \quad 1 = c^2.(1+1) \Rightarrow c^2 = \therefore \quad 1 = c^2 \frac{1}{2}$

57. if $y = y(x)$ is the solution of the differential equation $\frac{5+e^x}{2+y}\frac{dy}{dx} + e^x = 0$ satisfying $y(0) = 1$, then a value of $y(\log_e 13)$ is :

(1) –1 (2) 0 (3) 2 (4) 1

Ans. (1)

Sol. Given $\frac{dy}{2+y} = \frac{-e^x dx}{5+e^x}$

$\ell n(2 + y) = -\ell n(5 + ex) + \ell nC$

$y = \frac{C}{5+e^x} - 2$

$y(0) = 1 \quad \therefore \quad C = 18$

$y = \frac{18}{5+e^x} - 2$

$\therefore \quad y = (\log_e 13) = -1$

58. If $\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x + e^{-x})}dx = g(x)e^{(e^x + e^{-x})} + c$, where c is a constant of integration, then g(0) is equal to :

(1) 1 (2) e (3) e^2 (4) 2

Ans. **(4)**

Sol. $I = \int \left(e^{2x} + 2e^{x} - e^{-x} - 1\right) e^{\left(e^{x}+e^{-x}\right)} dx$

$I = \int \left(e^{2x} + e^{x} - 1\right) e^{\left(e^{x}+e^{-x}\right)} dx + \int \left(e^{x} - e^{-x}\right) e^{e^{x}+e^{-x}} dx$

$I = \int \left(e^{x} + 1 - e^{-x}\right) e^{e^{x}+e^{-x}+x} dx + e^{e^{x}} + ^{e^{-x}}$

$e^{x} + e^{-x} + x = du$

$(e^{x} - e^{-x} + 1)\, dx = du$

$I = e^{e^{x}+e^{-x}+x} + e^{e^{x}+e^{-x}} = e^{e^{x}+e^{-x}}(e^{x}+1)$

then g(x) e^{x} +1

g(0) = 2

59. If α is the positive root of the equation, p(x) = $x^2 - x - 2 = 0$, then $\lim_{x\to\alpha} \frac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to :

(1) $\frac{3}{2}$ (2) $\frac{3}{\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

Ans. (2)

Sol. P(x) = 0

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1 \therefore \alpha = 2$

Now $\lim_{x\to 2^+} \frac{\sqrt{1-\cos(x^2-x-2)}}{x-2} \Rightarrow \lim_{x\to 2^+} \frac{\sqrt{2\sin^2\left(\frac{x^2-x-2}{2}\right)}}{x-2}$

$\Rightarrow \lim_{x\to 2^+} \frac{\left|\sin\left(\frac{x^2-x-2}{2}\right)\right|}{x-2} \Rightarrow \lim_{x\to 2^+} \frac{\sqrt{2\sin^2\left(\frac{x^2-x-2}{2}\right)}}{x-2}$

$\lim_{x\to 2^+} \frac{\sqrt{2}\sin\frac{(x^2-x-2)}{2}}{\left(\frac{x^2-x-2}{2}\right)}, \frac{\frac{x^2-x-2}{2}}{(x-2)} \Rightarrow \lim_{x\to 2^+} \frac{1}{\sqrt{2}}, \frac{(x-2)(x+1)}{x-2} = \frac{3}{\sqrt{2}}$

60. If the co–ordinates of two points A and B are ($\sqrt{7}$, 0) and ($-\sqrt{7}$, 0) respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then PA + PB is equal to :

(1) 8 (2) 16 (3) 9 (4) 6

Ans. **(1)**

Sol. For ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1, a = 4, b = 3, e = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$

A and B are foci

then PA + PB = 2a = 2(4) = 8

61. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$ is :

(1) $\frac{25}{81}$ (2) $\frac{5}{27}$ (3) $\frac{25}{9}$ (4) $\frac{5}{9}$

Ans. (1)

Sol. $\therefore x^2 = |x|^2 = t$ let

$9t^2 - 18t + 5 = 0$

$(3t - 1)(3t - 5) = 0$

$|x| = \frac{1}{3}, \frac{5}{3}$

$\text{product of roots} = \frac{1}{3}\left(-\frac{1}{3}\right)\left(\frac{5}{3}\right)\left(\frac{-5}{3}\right) = \frac{25}{81}$

62. If the volume of a parallelopiped, whose conterminous edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}\ (n \geq 0)$, is 158 cu-units, then :

(1) n = 7 (2) n = 9 (3) $\vec{b}.\vec{c} = 10$ (4) $\vec{a}.\vec{c} = 17$

Ans. (3)

Sol. Volume of parallelepiped v = $\left|[\vec{a}\,\vec{b}\,\vec{c}]\right|$

$$v = \begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = \pm 158$$

$1(12 + n^2) - 1(6 + n) + n(2n - 4) = \pm 158$

$3n^2 - 5n - 152 = 0$ or $3n^2 - 5n + 164 = 0$ D < 0 (no real roots)

$n = 8, -\frac{19}{3} \Rightarrow n = 8$

then $\vec{b}.\vec{c} = 2 + 4n - 3n = 10$

$\vec{a}\vec{c} = 1 + n + 3n = 33$

63. If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point Q(0, –4), then PQ^2 is equal to :

(1) 29 (2) 48 (3) 21 (4) 36

Ans. (4)

Sol. Euqation $\frac{x^2}{5} + \frac{y^2}{4} = 1$ then $P\left(\sqrt{5}\cos\theta, 2\sin\theta\right)$

$(PQ)^2 = 5\cos^2\theta + 4(\sin\theta + 2)^2 = \cos^2\theta + 16\sin\theta + 20 = -\sin^2\theta + 16\sin\theta + 21$

$= (PQ)^2_{max} = 85 - 49 = 36, \quad \because \quad (\sin\theta - 8)^2 \in [49, 81]$

64. If (a,b,c) is the image of the point (1,2, –3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = r$, then a + b + c is equal to :

(1) 3 (2) –1 (3) 2 (4) 1

Ans. (3)

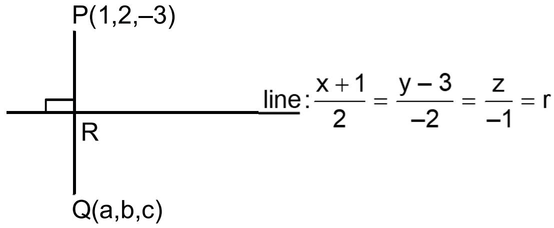
Sol. R(–1 + 2r, 3 – 2r, –r)

dr's of PR are (2 – 2r, –1 + 2r, –3 + r)

Then 2(2 – 2r) + 2(1 – 2r) + 1(3 – r) = 0

9 – 9r = 0 $\Rightarrow$ r = 1

R(1, 1, –1)



then a + 1 = 2 b + 2 = 2 c – 3 = –2

a = 1 b = 0 c = 1

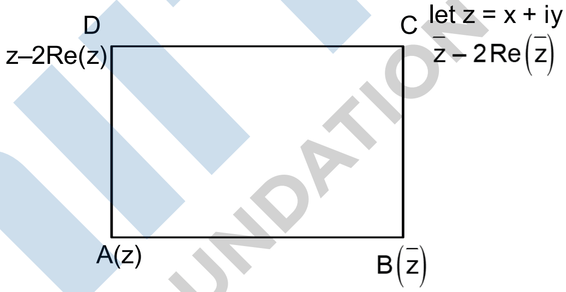
$\therefore$ a + b + c = 2

65. If the four complex numbers $z, \bar{z}, \bar{z} - 2\text{Re}(\bar{z})$ and $z - 2\text{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then |z| is equal to :

(1) 4 (2) 2 (3) $4\sqrt{2}$ (4) $2\sqrt{2}$

Ans. (4)

Sol. $\therefore$ length of side = 4



then $|z - \bar{z}| = 4$

|2iy| = 4

|y| = 2

also |z – (z – 2Re(z)| = 4

|2x| = 4 $\Rightarrow$ |x| = 2

$|z| = \sqrt{x^2 + y^2} = 2\sqrt{2}$

66. If the minimum and the maximum values of the function $f : \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \to R$, defined by

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

are m and M respectively, then the ordered pair (m, M) is equal to :

(1) (–4, 4) (2) $(0, 2\sqrt{2})$ (3) (–4, 0) (4) (0,4)

Ans. (3)

Sol. $C_2 \to C_2 - C_1$

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1 & 1 \\ -\cos^2\theta & -1 & 1 \\ 12 & -2 & -2 \end{vmatrix} = 4(\cos^2\theta - \sin^2\theta) = 4(\cos^2\theta),\ \theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$$

$f(\theta)_{max} = M = 0$

$f(\theta)_{min} = m = -4$

67. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to :

(1) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$
(2) $(x \wedge y) \vee (\sim x \wedge \sim y)$
(3) $(x \wedge \sim y) \vee (\sim x \wedge y)$
(4) $(x \wedge y) \wedge (\sim x \vee \sim y)$

Ans. (2)

Sol. Negation of $x \leftrightarrow \sim y$

$\equiv \sim(x \leftrightarrow \sim y)$

$\equiv x \leftrightarrow \sim (\sim y))$

$\equiv x \leftrightarrow y$

$\equiv (x \wedge y) \vee (\sim x \wedge \sim y)$

68. Let $\lambda \in R$. The system of linear equations

$2x_1 - 4x_2 + \lambda x_3 = 1$

$x_1 - 6x_2 + \lambda x3 = 2$

$\lambda x_1 - 10x_2 + 4x_3 = 3$

is inconsistent for :

(1) every value of λ
(2) exactly two values of λ
(3) exactly one positive value of λ
(4) exactly one negative value of λ

Ans. (4)

Sol. Here $D = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = (\lambda - 3)(3\lambda + 2)$

$D = 0 \quad \Rightarrow \quad \lambda = 3, -\frac{2}{3}$

$D_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix} = 2(3 - \lambda)$

For $\lambda = -\frac{2}{3}, D_1 \neq 0$

69. The mean and variance of 7 observations are 8 and 16, respectively. If five observation are 2,4,10,12,14 then the absolute difference of the remaining two observations is :

(1) 1
(2) 2
(3) 3
(4) 4

Ans. (2)

Sol. $\bar{x} = \frac{2+4+10+12+14+x+y}{7} = 8 \quad \Rightarrow \quad 42 + x + y = 56 \Rightarrow \quad x + y = 14$

$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$$16 = \frac{4+16+100+144+196+x^2+y^2}{7} - (8)^2$$

$$\Rightarrow \quad 16+64 = \frac{460+x^2+y^2}{7}$$

$\Rightarrow \quad 560 = 460 + x^2 + y^2 \quad \Rightarrow \quad x^2 + y^2 = 100$ .....(2) $\quad \Rightarrow \quad xy = 48$

$(x - y)^2 = (x + y)^2 - 4xy = 4$

$|x - y| = 2$

70. The value of $\int_{-\pi/1}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$. is :

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{3\pi}{2}$ (4) π

Ans. (2)

Sol. $I = \int_{-\pi/1}^{\pi/2} \frac{1}{1+e^{\sin x}} dx.\ I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$

$$I = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{1+e^{\sin x}} dx \begin{cases} \text{Replace} \\ x \to (a+b+x) \end{cases}$$

$$\int_a^b \left(F(X)\,dx = \int_c^b f(a+b+x)\,dx \right.$$

$$2I = \int_{-\pi/2}^{\pi/2} 1dx \Rightarrow I = \frac{1}{2}\int_{-\pi/2}^{\pi/2} dx$$

$$I = \frac{1}{2}[x]_{-\pi/2}^{\pi/2} \Rightarrow I = \frac{\pi}{2}$$

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

71. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is _________.

Ans. (240.00)

Sol. SYLLABUS

S-2, L-2, A, B, Y, U

$$\text{Required} = {}^2C_1 . {}^5C_2 . \frac{4!}{2!} = 2.10.\frac{24}{2} = 240$$

72. The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is_________

Ans. (13.00)

Sol. $T_{r+1} = {}^{22}Cr\,(x^m)^{22-r}\,x^{-2r}$

$T_{r+1} = {}^{22}C_r\,x^{m(22-r)-2r}$

$22m - mr - 2r = 1$

$22m - 1 = r\,(m + 2)$

$$r = \frac{22m - 1}{m + 2}$$

$$r = \frac{22m + 44 - 45}{m + 2}$$

$$r = 2 - \frac{3.35}{m + 2}$$

so possible value of m = 1,3, 7, 13, 43

but ${}^{20}C_r = 1540$

only possible condition is m = 13

23. If the line, 2x – y + 3 = 0 is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines 4x – 2y + α = 0 and 6x – 3y + β = 0, respectively, then the sum of all possible values of α and β is_________

Ans. (30.00)

Sol. 2x – y + 3 = 0 .....(i)

$4x - 2y + \alpha = 0 \quad \Rightarrow \quad 2x - y + \frac{\alpha}{2} = 0$..........(ii)

$6x - 3y + \beta = 0 \quad \Rightarrow \quad 2x - y + \frac{\beta}{3} = 0$..........(iii)

$$d_1 = \frac{\left|\frac{\alpha}{2} - 3\right|}{\sqrt{2^2 + 1^2}} = \frac{1}{\sqrt{5}} \Rightarrow |\alpha - 6| = 2 \Rightarrow \alpha - 6 = 2, -2 \Rightarrow \alpha = 8, 4$$

$$d_2 = \frac{\left|\frac{\beta}{3} - 3\right|}{\sqrt{2^2 + 1^2}} = \frac{1}{\sqrt{5}} \Rightarrow |\beta - 9| = 6 \Rightarrow \beta - 9 = 6, -6 \Rightarrow \beta = 15, 3$$

Sum of all values of α and β = 30.

74. Let $f(x) = x.\left[\frac{x}{2}\right]$, for $-10 < x < 10$, where [t] denotes the greatest integer function. Then the number of points of discontinuity of f is equal to __________

Ans. (08.00)

Sol. $-5 < \frac{x}{2} < 5$

$$\Rightarrow \quad \left[\frac{x}{2}\right] = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4,$$

Hence, function is discontinues at = –4, –3, –2, –1, 1, 2, 3, 4

Number of values is 8.

75. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is __________

Ans. (11.00)

Sol. P(at least 2 show 3 or 5) = $^4C_2.\left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)^2 + {^4C_3}\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right) + {^4C_4}\left(\frac{2}{6}\right)^4$

$$= \frac{384 + 128 + 16}{6^4} = \frac{11}{27}$$

n = 27

$\therefore$ expectation of number of times = np

$$= 27.\frac{11}{27} = 11$$